Politehnica University of Bucharest
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New relationships and methods of determination with high precision of the coordinates of the maximum power point of a photovoltaic cell (Summary)

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PREFACE

This thesis was carried out during the doctoral studies within the Doctoral School of Electrical Engineering, under the scientific coordination of Mr. prof. Univ. dr. eng. Aurelian Crăciunescu.

I would like to thank the doctoral supervisor, Mr. Prof. Univ. dr. eng. Aurelian Crăciunescu, for the support provided throughout the period of conducting doctoral studies but also for the support provided in the elaboration of this work.

I want to thank my family members, who understood and supported me in my efforts to complete this work, an important step during my professional career.

Finally, I would like to thank all the teachers who have contributed to my educational and professional training over time.

Mircea Taciuc

TABLE OF CONTENTS (ORIGINAL THESIS)

Note: *The numbering of the references and relationships in the original thesis are used in the content of this abstract.*

Keywords: *maximum power point coordinates, intersection method, single diode model, applications of the W Lambert function, explicit form of PV cell characteristic, variable electronic load*

INTRODUCTION

The maximum power point is the most important point on the operating characteristic of a photovoltaic cell. The dynamics of the real operating conditions of a photovoltaic cell is fully reflected on the position of this point, being amplified by a nonlinear correlation. Due to this non-linear correlation, but also due to the dynamism of the operating conditions, the precise identification of the coordinates of the maximum power point turns into a difficult task.

Determining the position of the maximum power point for photovoltaic cells is a topic for many works in the literature. Most works deal with this subject based on the implementation of algorithms and only a small part of the works deal with this subject, based on an analytically aproach, by starting from the theoretical models of the photovoltaic cell.

Approaches based on algorithms

The most studied "classic" algorithms are the Perturb and Observe (PO) or Incremental Conductance (IC). The modern algorithms are implementations derived from EC-Evolutionary Computation algorithms such as Differential Evolution (DE).), Genetic Algorithm (GA), Particle Swarms Optimization (PSO).

Analytically aproaches

From the point of view of the theoretical analysis, the difficulty of identifying of the position of the maximum power point is due to the presence of implicit-transcendent equations in the mathematical models that describe the photoelectric phenomenon that occurs in photovoltaic cells.

A set of relations for calculating of the maximum power point coordinates will very clearly define the position of this point on the characteristic of the photovoltaic cell. Identifying such relationships can also significantly simplify the reverse issue: determining of the main parameters of the mathematical model used in the description of photovoltaic cells.

Most works in the literature deal with the identification of some mathematical relationships for estimating maximum power through based on fill factor (FF). The disadvantage of this approach is that the value of the fill factor provides clues only on the value of maximum power and efficiency of the photovoltaic cell but does not provide clear indications on the position/coordinates of the maximum power point on the voltage-current characteristic of the photovoltaic cell. In the literature are several relatively precise relations for estimating of the fill factor, of which the most used, for the three-parameter model, is the "Green relation" (5.1.37).

PURPOSE AND OBJECTIVES OF THE WORK

The main purpose of this work is to identify explicit relationships for the direct (non-iterative) calculation of the maximum power point coordinates for silicon-based single-junction photovoltaic cells.

In this work are proposed several sets of mathematical relations for the high precision calculation of the coordinates of the maximum power point. Relationships are presented for the three-parameter model (3P- "ideal model"), for the four-parameter model (4P) and for the five-parameter model (5P).

Presented solving methods are based on two types of approaches, one aproach is based on a series of analytical-geometric properties of the voltage-current characteristic of photovoltaic cells and the other aproach is based on properties of Lambert's W function useful in solving transcendental equations.

For the five-parameter model, a set of relationship for the calculation of the coordinates of the maximum power point is identified and proposed. These relationships where identified based on an original method presented in this work ("intersection method"). The proposed method is based on the observation of a property related to the point of intersection of the tangent lines to the characteristic voltage-current curve of photovoltaic cells.

As a secondary purpose, but no less important, this work aims to present the design, and implementation of an experimental system designed to "raise" the voltage-current characteristic for photovoltaic panels in real operating conditions.

The experimental system was installed on the roof of the Faculty of Electrical Engineering Bucharest and following its operation were "collected" a large number of voltage-current characteristics for a wide range of operating conditions (solar radiation intensity and operating temperature) of the tested panels.

STRUCTURE OF THE THESIS

The work is structured in ten chapters. The work is divided into two general parts: a part of theoretical study and a practical part.

The theoretical study part is dedicated to the analysis of the maximum power point of photovoltaic cells. This part is oriented mainly towards to the identification of several mathematical relations that ensure the direct calculation of the coordinates of the most important point on the operating characteristics of a photovoltaic cell: *the maximum power point*. The study starts from the three models described in the introductory part and ends with the identification of several relations for the coordinates calculation of the maximum power point for each of the three studied models.

The practical part is dedicated to the description of the design and implementation of an experimental system, fully automated, which is used to "collect" the voltage-current characteristics for photovoltaic cells/panels. The realization of such an experimental system resulted from the natural need to create a database composed of voltage-current characteristics obtained for a wide range of real operating conditions of photovoltaic cells/panels. The experimental data thus obtained can be very useful in understanding the operation of photovoltaic cells in real conditions but also in checking/validating various mathematical relationships for modeling photovoltaic cells.

Chapter 1

Physical models of the photovoltaic cell as reference models for calculating of the technical characteristics of photovoltaic cells

This chapter is an overview of the characteristic equations of the photovoltaic cells and their parameters, which are derived from the well-known single-diode model. The equations presented in this chapter represent the basis of the theoretical study developed in this work.

Chapter 2

Normalized forms of the characteristic equations

In this chapter is proposed and presented an original approach (new in the literature) on rewriting the characteristic equations of photovoltaic cells in a normalized form.

The rewriting of the characteristic equations in a normalized form is based on the observation that the maximum values of voltage (V_{oc}) and current (I_{sc}) on the operating characteristic of a photovoltaic cell are well defined for each set of values of external operating parameters (radiation intensity solar and operating temperature). Based on this observation, normalized forms are proposed for all three models addressed in this work.

The use of the normalized forms has several advantages, some of which will be highlighted in the following chapters of this work.

Normalized characteristic equations have a simpler form with a reduced number of terms, which makes them easier to use for mathematical operations, such as derivation and/or integration, applied to characteristic equations.

The normalized forms also have advantages in terms of graphical representations of characteristic curves. The graphical representations of these curves are reduced to the first unitary quadrant which helps to make them easier to interpret. Also, the representation in the first unitary quadrant could help the easy development of the rewriting of the characteristic equations in a polar type form (subject not yet addressed in the literature).

Another advantage of normalized forms is that are based on two parameters that can be easily obtained experimentally by direct measurements (I_{sc} and V_{oc}), which makes them useful for processing of the experimental data.

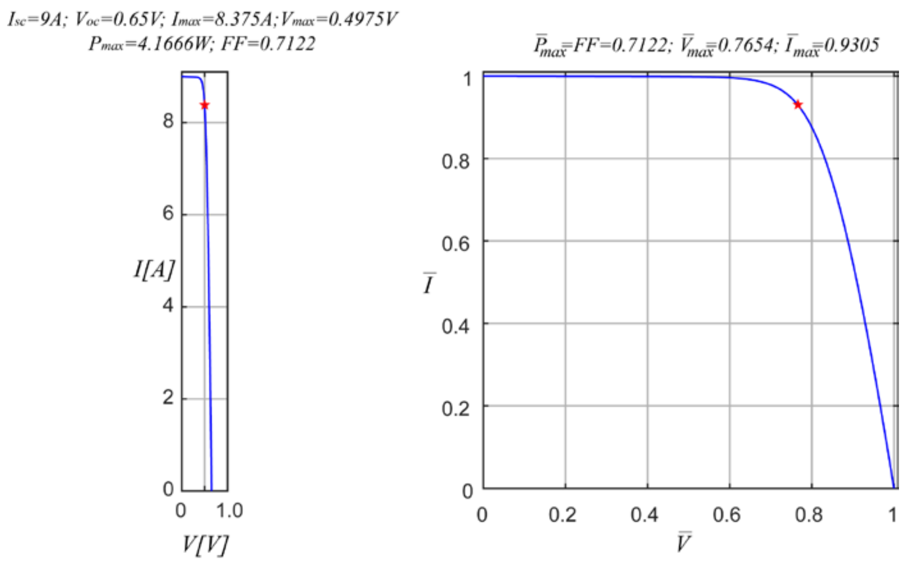
In the continuation of this work, the references to these forms of photovoltaic cell equations will be made under the generic name of "normalized forms".

For the rewriting in normalized form of the photovoltaic cells equations the following changes of variables are proposed:

$$\begin{cases} \bar{V} = \dfrac{V}{V_{oc}} \\ \bar{I} = \dfrac{I}{I_{sc}} \end{cases}$$

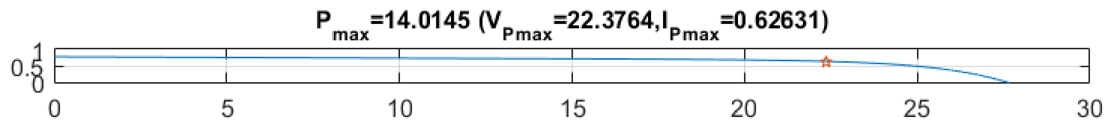
The parameters of the equations expressed in normalized forms are also used [3]:

No load operating voltage in normalized form *(v_{oc})*	$v_{oc} = \dfrac{V_{oc}}{m \cdot V_T}$
Normalized series resistance *(r_s)*	$r_s = \dfrac{R_s}{R_{ch}} = \dfrac{R_s}{\left(\dfrac{V_{oc}}{I_{sc}}\right)}$
Normalized parallel resistance *(r_p)*	$r_p = \dfrac{R_p}{R_{ch}} = \dfrac{R_p}{\left(\dfrac{V_{oc}}{I_{sc}}\right)}$
The characteristic resistance of the photovoltaic cell *(R_{ch})*	$R_{ch} = \dfrac{V_{oc}}{I_{sc}}$

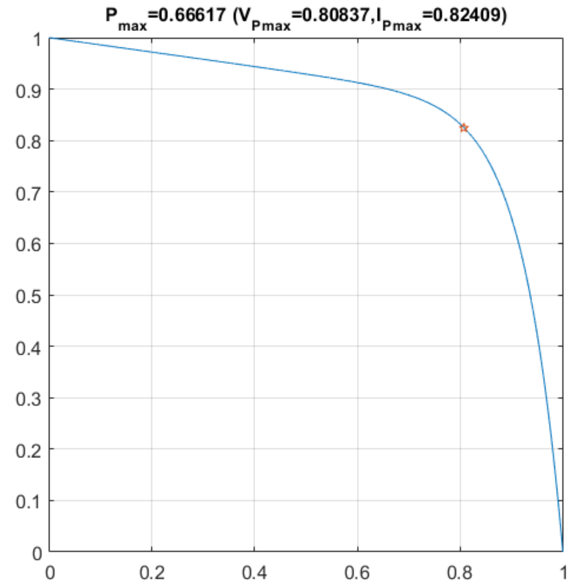


a) *representation in non-normalized form* ***b)*** *representation in normalized form*

Fig. 2.1. *Graphical representation of the I-V characteristic for a photovoltaic cell a single cell exposed to high intensities of solar radiation (Representation with the same scale on both axes)*



***a)** representation in non-normalized form*



***b)** representation in normalized form*

***Fig. 2.2**. Graphical representation of the I-V characteristic for a photovoltaic cell a single cell exposed to low intensities of solar radiation (Representation with the same scale on both axes)*

Chapter 3

Explicit forms of the characteristic equations
Solution using Lambert's W function

With the exception of the three-parameter model (3P), the use of characteristic equations in the general form (1.2.1), (1.2.3) is cumbersome due to their implicit-transcendent form. The analysis of the characteristics based on these equations involves using of iterative processes with a relatively complex logic scheme and which require a long processing time. The inverse problem, the parameters determination in the conditions in which experimental data are available, is even more laborious requiring a very long processing time, especially if many sets of experimental data are analyzed.

In the first part of this chapter, starting from equations (1.2.1) (1.2.3), an explicit form of the characteristics *I-V* and *V-I* is determined. The mathematical demonstration, presented in this chapter, is based on the use of Lambert's W function in solving of implicit exponential or logarithmic equations. The demonstration starts from the normalized form, of the characteristic equation of the five-parameter model (2.2.22) proposed in this work, and highlights the ease of use of the normalized forms in mathematical “processing”.

In the second part of this chapter, a series of approximations are proposed that can be applied to the explicit equations obtained without significantly affecting their accuracy in estimating of the characteristic curves. The purpose of applying these simplifications is to reduce the number of terms of the characteristic equations and to make their use much easier. Following the application of the proposed approximations, three simplified forms of the basic explicit equation (SF1) (SF2) (SF3) are obtained.

In the final part of the chapter is presented an analysis of the effect of the adopted simplifications on the accuracy of the voltage-current characteristic determination. The analysis is performed based on numerical simulations and the results are presented by comparing them with the numerical simulations obtained based on the initial unsimplified form.

These simulations showed that two of the proposed simplified forms had a deviation of less than 15ppm. The third proposed simplified form has a determination deviation of less than 0.2% which makes it sufficiently accurate for any type of evaluation of the photovoltaic cells characteristics.

The explicit normalized forms obtained for the characteristic equations of the photovoltaic cell are presented below.

$$\begin{cases} \bar{I}_{(\bar{V})} = A - \dfrac{W\left\{B \cdot C \cdot e^{(A \cdot C)}\right\}}{C} \\ A = \dfrac{1}{r_s + r_p} \cdot \left[\left(r_s + r_p - 1\right) \cdot \bar{E}_{rs} + 1 - \bar{V}\right] \\ B = \dfrac{r_s + r_p - 1}{r_s + r_p} \cdot \bar{E}_{rs} \cdot e^{v_{oc}\left(\bar{V} - 1\right)} \\ C = r_s \cdot v_{oc} \end{cases} \quad (3.2.15)$$

$$\begin{cases} \overline{V}_{(\bar{I})} = A' - \dfrac{W\left\{B' \cdot C' \cdot e^{(A' \cdot C')}\right\}}{C'} \\ A' = (r_s + r_p - 1) \cdot \overline{E}_{rs} - \bar{I} \cdot (r_s + r_p) + 1 \\ B' = (r_s + r_p - 1) \cdot \overline{E}_{rs} \cdot e^{v_{oc} \cdot (r_s \cdot \bar{I} - 1)} \\ C' = v_{oc} \end{cases} \tag{3.2.16}$$

Where, for $\overline{E}_{rs}$, the (2.2.18) notation was used:

$$\overline{E}_{rs} = \frac{1}{1 - \dfrac{1}{e^{v_{oc} \cdot (1 - r_s)}}}. \tag{3.2.16'}$$

Chapter 4

Numerical methods for calculating the values of Lambert's W function

The advantage of the explicit forms presented in *Chapter 3* is diminished by the presence of the Lambert's W function in these relationships. Since Lambert's W function is not an algebraic function, the exact calculation of its values cannot be performed on the basis of computational relations with finite number of operations. For the approximate calculation, but with very good precision, of the values of this function, implementations of iterative / recursive numerical methods are necessary.

The objective of this chapter is to identify an iterative numerical method that allows the calculation as quickly and accurately as possible of the values of Lambert's W function.

In this chapter a function of own conception is proposed and presented (*Q_lambertw*), for the fast and precise calculation of the values of the W function of Lambert. The proposed subroutine was designed as a Matlab script and is based on an iterative algorithm derived from the Halley method adapted for the Lambert's W function [14].

In the first part of this chapter is made a presentation of two iterative algorithms derived from the Newton-Raphson method and the Halley method that can be used to numerically calculate the values of the Lambert's W function.

In this chapter is proposed and presented an iterative algorithm adapted for calculating the values of the W function of Lambert (for the positive values of the branch W_0). Also, a set of initial values for the implemented algorithm is proposed and the optimization criteria that have been adopted to increase the processing speed are presented.

Based on the proposed algorithm, a function (*Q_lambertw*) was implemented in MatLab, which ensures the extremely fast and very precise processing of the values of Lambert's W function.

The final part of the chapter presents the results of the evaluation of the proposed function (*Q_lambertw*) in terms of processing speed and accuracy of calculation of the Lambert's W function values. The data obtained are presented in comparison with the results obtained with the "*lambertw*" function which is included in the standard package of Matlab functions. Following this comparison, it is shown that the proposed function is hundreds of times faster and at least as accurate with the "*lambertw*" function that is included in the standard Matlab function package.

Based on the proposed function (*Q_lambertw*), a set of MatLab functions has been developed that ensures extremely fast processing of characteristics for photovoltaic cells. These functions ensure the evaluation of 1000 characteristics, each composed of 500 points, in less than 0.5msec. (MatLab R2016a installed on an Intel Core-i3-Gen4 platform).

Chapter 5

Relationships for determining the coordinates of the maximum power point. Three-parameter model (3P)

In this chapter, four sets of explicit relationships are determined for the direct (non-iterative) calculation of the coordinates of the maximum power point for the three-parameter model (3P - "ideal model"). The proposed relations, are determined starting from the normalized forms of the characteristic equations of the analyzed model.

The determination of the relations is made by two different approaches: one based on the grapho-analytical and the other based on the properties of the Lambert's W function. Both approaches are applied to solve at least one of the equations resulting from the zeroing of the derivative of the power functions obtained from the voltage-current characteristics.

The evaluation of the accuracy, for each determined set of relationships, is performed by numerical simulations using MatLab. The simulation data are presented by reporting the results obtained, based on the proposed relationships, to the results obtained by numerically solving of the basic equations using the MatLab symbolic solver "vpasolve".

In order to highlight the exceptional accuracy of the proposed relationships, the accuracy of the most used relationship in the literature was also evaluated, for estimating the fill factor (FF): “Green relationship” (5.1.37).

The comparative analysis indicates that all four proposed relationships are more accurate than the "Green relationship". The proposed relationships have the great advantage of identifying the point of maximum power by clearly defining the two coordinates of this important point.

At least two of the relationships proposed in this work (5.1.34) (5.1.35) can be considered as the most accurate in the literature regarding the explicit-analytical determination of the coordinates of the maximum power point for the three-parameter model.

$$\begin{cases} \bar{I}_{\max} \cong 1-\left(\dfrac{1}{v_{oc}+1}\right)^{\left(\frac{v_{oc}}{v_{oc}+1}\right)} \\ \bar{V}_{\max} \cong 1-\dfrac{\ln(v_{oc}+1)}{v_{oc}+1} \end{cases} \qquad (5.1.35)$$

Chapter 6

Relationships for determining of the maximum power point coordinates.
Four-parameter model (4P)

In this part of the work, three sets of high-precision relationships are proposed for estimating of the maximum power point coordinates for the solar cells. The proposed relations have an explicit form and have the advantage that they allow the direct calculation of the coordinates of the maximum power point or of the fill factor.

The determination of these relations is done by three different methods: solving based on the properties of the Lambert's W function, solving by approximating of the parametric surface defined by the solutions of the studied equation and solving by reducing of the iterative Newton-Raphson method to a single iteration.

The evaluation of the accuracy for each of proposed relationships was performed by numerical simulations using MatLab. The simulation data are presented by reporting the results obtained, based on the proposed relationships, to the results obtained by solving the basic equations numerically using the symbolic solver “vpasolve” implemented in MatLab.

Among the relations proposed in this chapter the relation (6.1.44), characterized by a simple form with a relative error in determining the current coordinate of the maximum power point better than 2•10^{-4} and the relation (6.1.46) which although has a relatively complex form is characterized by a relative error in determining the current coordinate of the maximum power point (for the four-parameter model) less than 2•10^{-6} (2ppm).

$$\bar{I}_{\max(rs)} \cong 1.0044 - \frac{1.68 \cdot r_s^2 + 1.22}{v_{oc} \cdot (1 - 1.78 \cdot r_s) + 14.4 \cdot r_s^2 + 0.4} \quad (6.1.44)$$

$$\begin{cases} \bar{I}_{\max(rs)} \cong 1 - \dfrac{1 + (1 - X_0) \cdot [1 + 2 r_s v_{oc} (1 - X_0)]}{2 + v_{oc} - 2 r_s v_{oc} (2X_0 - 1) + \ln(1 - X_0)} \\ X_0 = 1.0044 - \dfrac{1.68 \cdot r_s^2 + 1.22}{v_{oc} \cdot (1 - 1.78 \cdot r_s) + 14.4 \cdot r_s^2 + 0.4} \end{cases} \quad (6.1.46)$$

$$\bar{V}_{\max(rs)} = \bar{I}_{\max(rs)} \cdot \left[r_s + \frac{1}{v_{oc} \cdot \left(1 - \bar{I}_{\max(rs)}\right)} \right] \quad (6.1.14)$$

Chapter 7

A new method for estimating of the maximum power point coordinates for the case of the five-parameter model (5P).
The intersection method.

In this chapter a new method is proposed, generically called the intersection method, to identify the coordinates of the maximum power point for the five-parameter model (5P).

Following the study of many experimental data sets, obtained for various types of photovoltaic cells/panels, a correlation was identified between the position of the maximum power point and the point defined by the intersection of tangents, at the characteristic curve, in *(0, I_{sc})* and *(V_{oc}, 0)* points. The analysis showed that the point of maximum power is very close to a point *X (V_x, I_x)* defined by the intersection between *I-V* curve and a line *(d)* which is determined by the following two points (Fig. 7.1.3-6.1.5):

- Point of origin (I = 0, V = 0);
- Point *M (I_M, V_M)*, which is determined by the intersection of tangents in the *Isc* and *Voc* points, at the *I-V* curve.

The observation of this correlation led to a grapho-analytical approach, following which a set of relationships was determined, relations on the basis of which the coordinates of the maximum power point can be estimated.

The relationships proposed in this chapter (7.2.34a, b) can be used in estimating the maximum power point coordinates with an accuracy of better than 3% for the voltage coordinate *(V_{max})* and with an accuracy of more than 4% for the coordinate in current *(I_{max})*.

Based on these relations it is possible to estimate the maximum power, for the five-parameter model (5P), with an accuracy better than 1%.

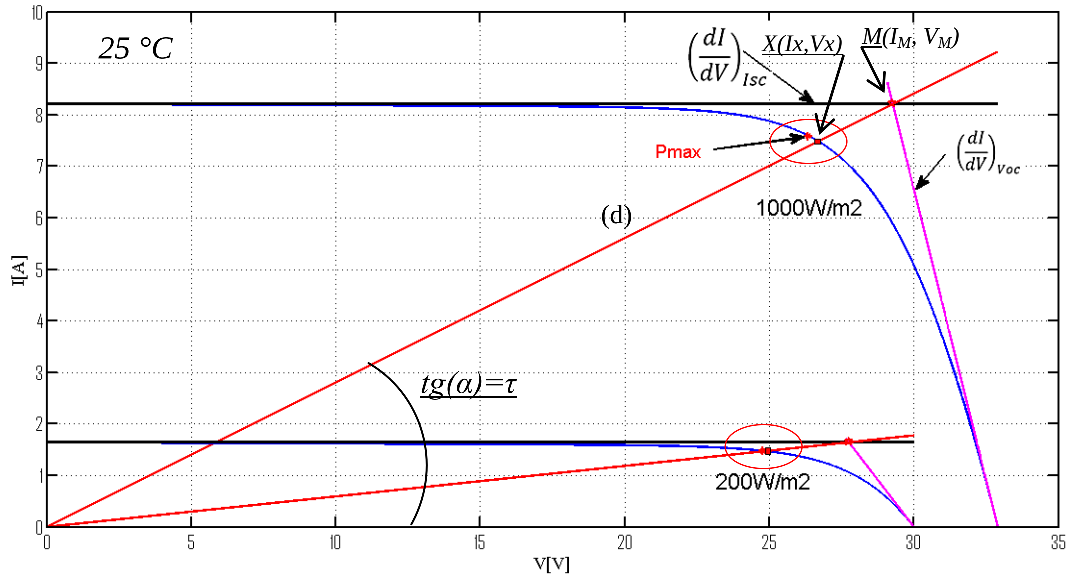


Fig. 7.1.3. *Intersection point for 1000W/m^2 si 200W/m^2*

$$\begin{cases} \bar{I}_X \cong \bar{I}_{\max} \cong \dfrac{v_{oc}\cdot(r_p-1)+1}{r_p\cdot v_{oc}}\cdot\left\{1-\dfrac{0.577}{v_{oc}-1}\cdot\ln\left[0.75\cdot\left(1-\dfrac{1}{r_p}\right)\cdot(v_{oc}-1)\right]\right\} & \text{(7.2.34a)} \\ \bar{V}_X \cong \bar{V}_{\max} \cong \dfrac{v_{oc}\cdot(1-r_s)-1}{v_{oc}}\cdot\left\{1-\dfrac{0.577}{v_{oc}-1}\cdot\ln\left[0.75\cdot\left(1-\dfrac{1}{r_p}\right)\cdot(v_{oc}-1)\right]\right\} & \text{(7.2.34b)} \end{cases}$$

Chapter 8

Experimental system for "raising" of the characteristic of photovoltaic panels

Usually, photovoltaic cell/panel manufacturers provide values of the main points of the characteristic I-V, values obtained in laboratory conditions (standard test conditions - STC and / or NOCT). These data provide indications of the I-V characteristic which are overestimated relative to the behavior of photovoltaic cells/modules in the real operating conditions.

This chapter presents a detailed description of an experimental system, designed and built, which can be used to "raise" the characteristic current-voltage curves for photovoltaic panels in real operating conditions. The experimental system was mounted and tested on the roof of the Faculty of Electrical Engineering.

The relatively short “raise” time of the I-V characteristics (1.5-2 sec.) makes the system able to collect the characteristics even in variable sky conditions for which the intensity of the solar radiation registers reasonable variations (<20-30 W/m^2/s). The system collects weather data (solar radiation intensity, ambient temperature, wind speed) and panel temperatures which allows an accurate assessment of the variation over the time of these quantities.

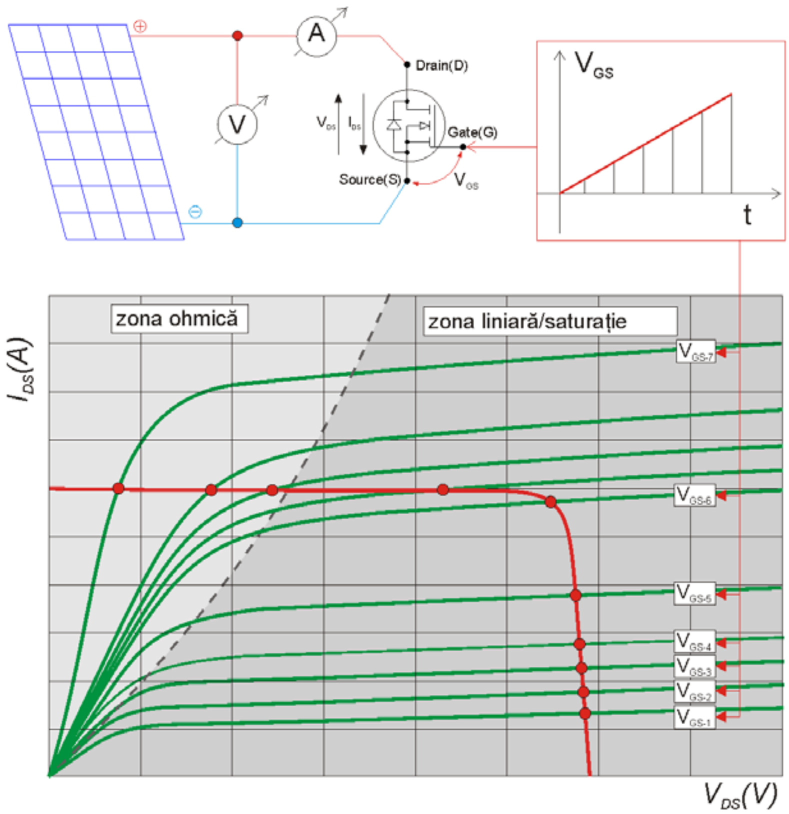


Fig. 8.3.1. *The principle of the electronic load resistance converter made with MOSFET type transistor*

- *With red line - characteristic I-V of the photovoltaic cell*
- *With green lines the I-V characteristics of the transistor for different control voltages (VGS)*

The control of the experimental system is performed by a software (own conception). Based on this software the system works completely autonomously ensuring the automatic collection, processing and saving of the collected data at predetermined time intervals.

The main functions provided by the system control software are:

- Controls the system of load resistance variation at predetermined time intervals and start the data acquisition from the voltage and current sensors. The start of these two stages is achieved by generating a pulse that will start the slope generator (presented in the previous chapter). The collection of the *I-V* data is automatically stopped after a number of a 10 consecutive zero values *(0.01V-0.05V)* of the voltage at the terminals of the photovoltaic panel. The measurement of a 10 consecutive zero values of the voltage at the terminals of the photovoltaic panel indicates that the transistors are completely open (load resistance is minimal) and the system has reached the point of short-circuit traversing the entire characteristic of the panel, from the open circuit voltage point $(V = V_{OC}, I = 0)$ to the short-circuit point $(V = 0, I = I_{SC})$;
- Continuously collects values from ambient and temperature sensors mounted on photovoltaic panels;
- Performs the mediation of values from environmental and temperature sensors;
- Writes to files at predetermined time intervals. Ambient and temperature data files include time stamp in ZZ-LL-HH: MM: SS (Day-Month-Hour Minute: Second) format. Files containing the data collected for characteristics I-V additionally include the time stamp (in milliseconds) of the purchased values;
- Provides the interface for the entire experimental system by displaying the various parameters of the system (Fig. 8.4.3):
    - Allows switching system control from automatic to manual;
    - Provides visualization (at 10 sec intervals) of mediated values of solar radiation intensity, ambient temperature and panel temperature.

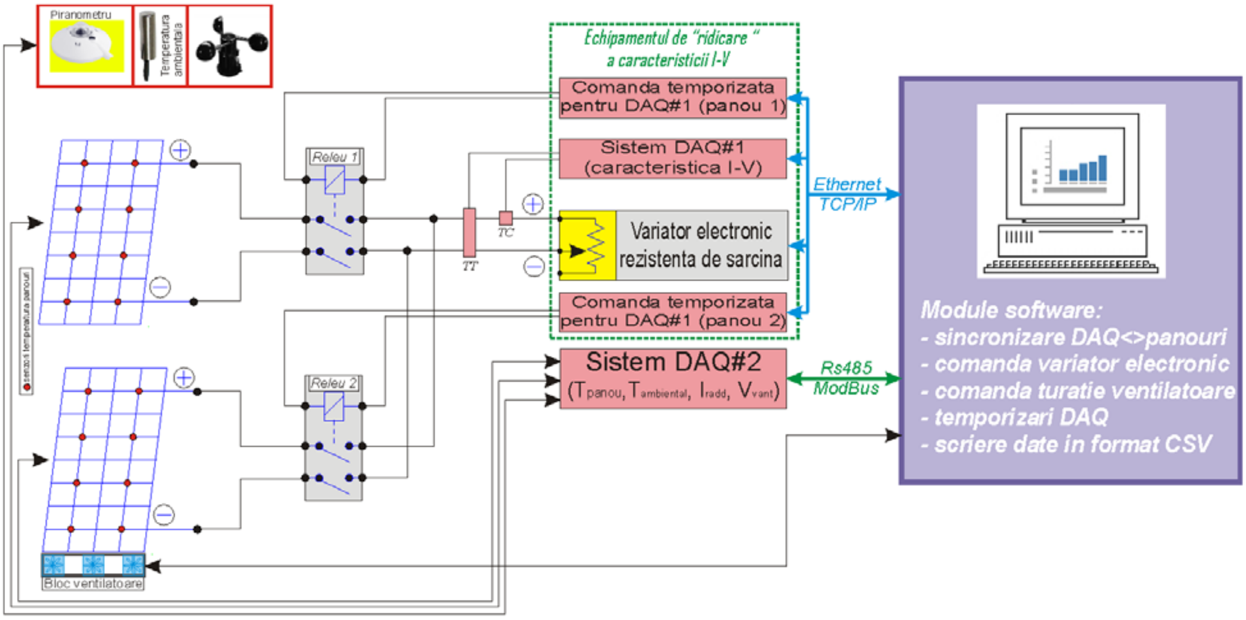


Fig. 8.4.1. *Functional schematic and structure of the experimental system*

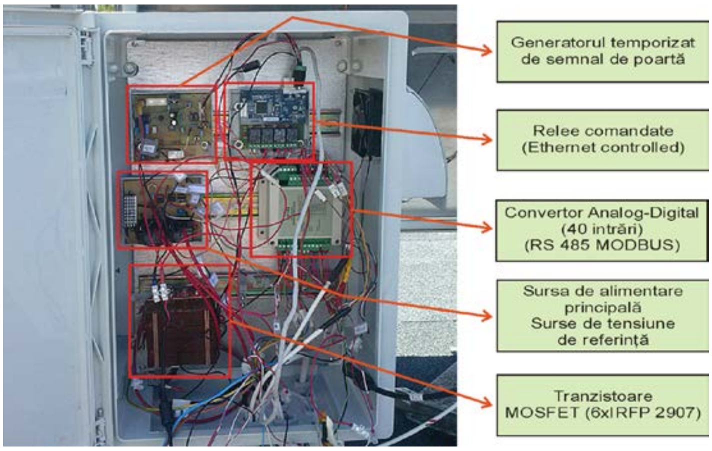


Fig. 8.4.2. *The electronic part of the experimental system*

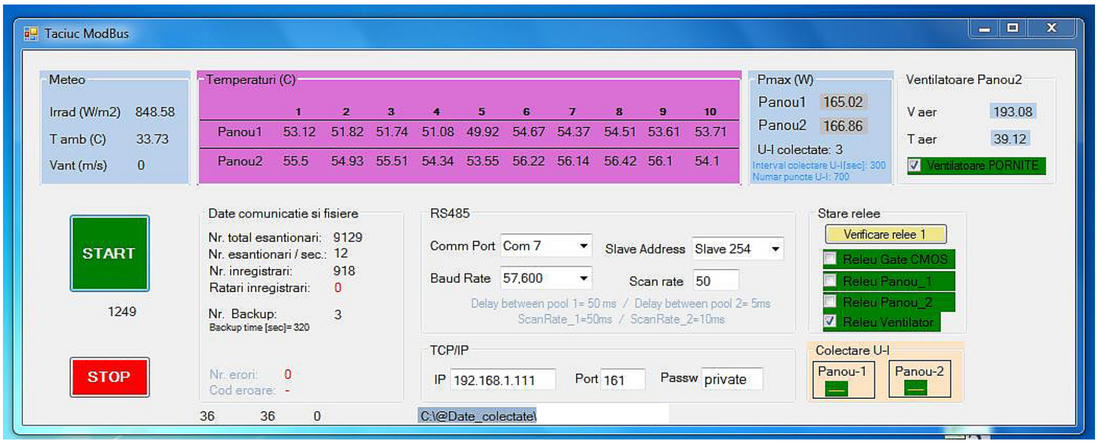


Fig. 8.4.3. *Interface with the experimental system*

Images with the experimental system installed on the roof of the Faculty of Electrical Engineering

Chapter 9
Analysis of the experimental data

In this chapter, a series of criteria for eliminating of the erroneous data and the way of processing of the obtained experimental data is presented. The criteria for "critical analysis" and data processing are set out specifically for the experimental system presented in Chapter 8.

The experimental system presented in the previous chapter allowed to obtain a large number of characteristic curves valid for a very wide range of operating conditions. Several sets of experimental data obtained are presented in Annex V and Annex VI for example.

In Fig. 9.1.1. some of the collected I-V characteristics at different times of the same day are presented. From these diagrams, it can be seen that the system collects a large number of points per characteristic. The relatively short time of the acquisition process (1-2 sec.) makes the system suitable for measurements even when the solar radiation is variable.

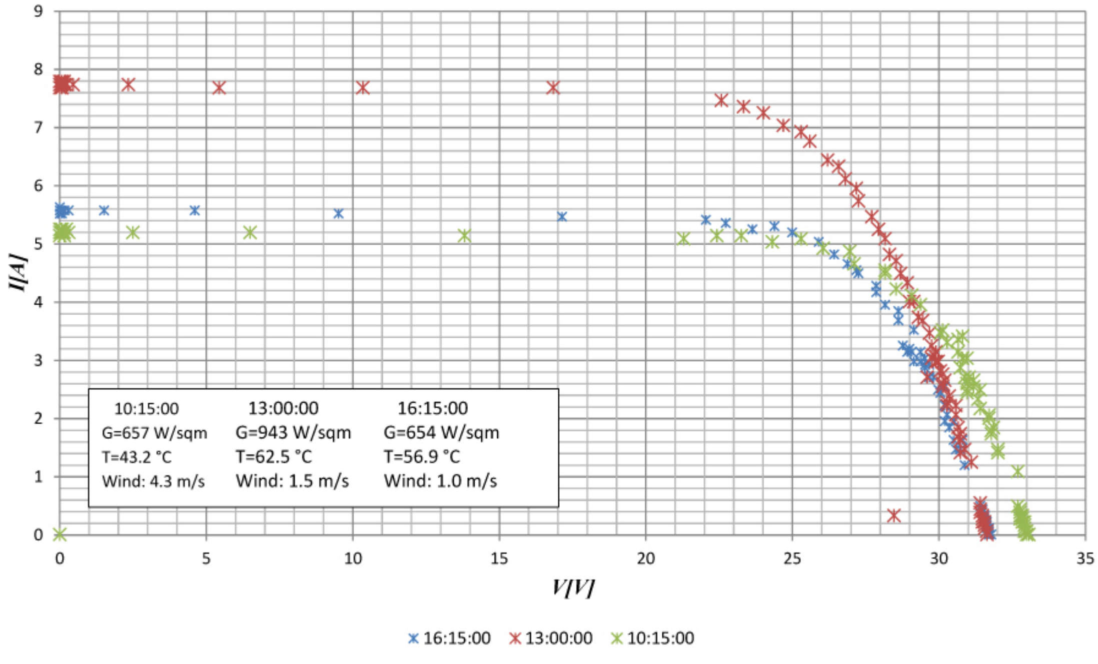


Fig. 9.1.1. *Example of I-V charactristics "raised" at various times of the same day*

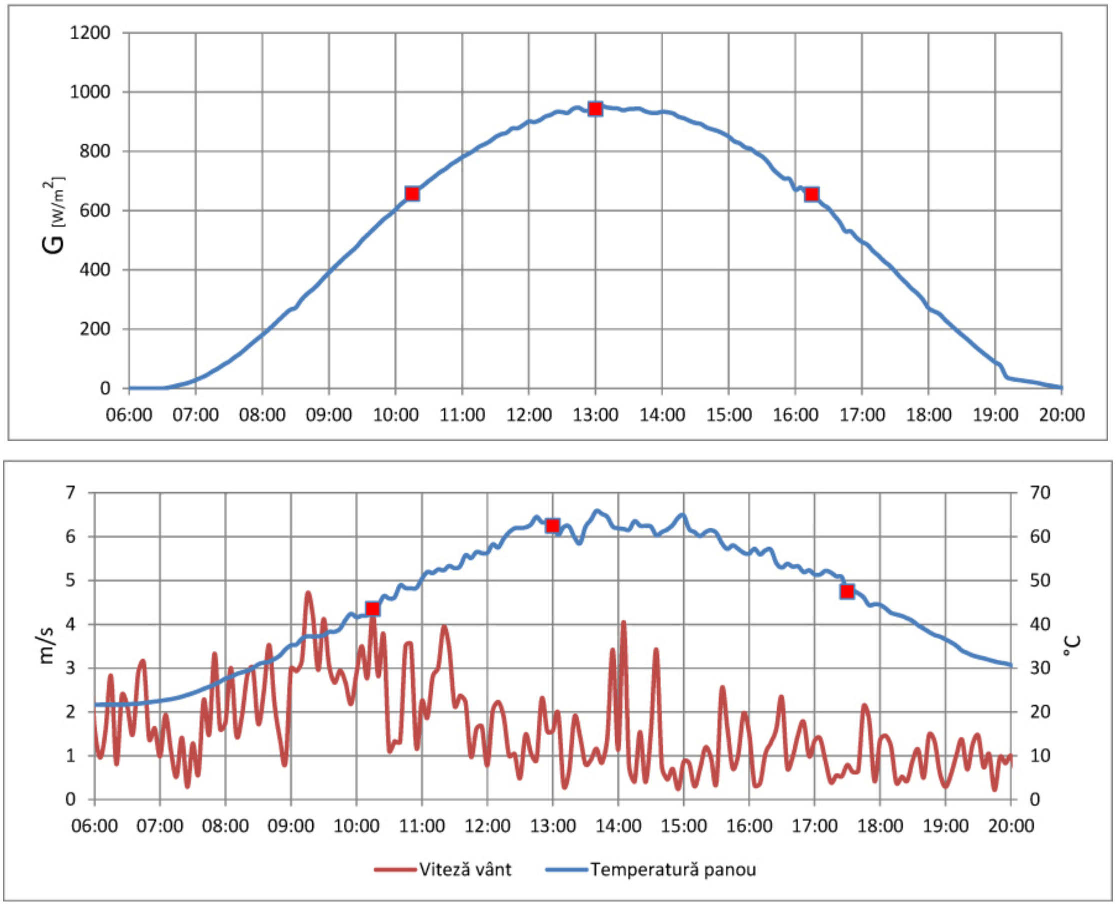


Fig. 9.1.2

(a) Variation in the intensity of solar radiation during the day (values averaged over 5 min.)
The points marked in red are detailed in Figs. 9.1.3
(b) Panel temperature variation during a day (values averaged at 5 min.)
The points marked in red are detailed in Figs. 9.1.4

The experimental system ensures by the design/construction a clear succession, in time, of the data acquisition. Because the system achieves a decreasing variation in load resistance over time, the data collected must comply with the following monotony:

- *Voltage* During the process of "raising" of the I-V characteristic, the voltage values decreases from the maximum value *(V_{oc})* to zero; $V \downarrow [Voc, 0]$;
  The first criterion for refining the data is thus obtained:
  Elimination of the values for which are recorded increasing voltage values: $[x_i - x_{(i+1)}] < 0$
- *Curentul* During the process of "raising" the characteristic I-V, the value of the current increases from zero to the maximum value *(Isc)* $I \uparrow [0, Isc]$;
  The second data refining criterion is thus obtained:
  Elimination of the values for which are recorded decreasing current values: $[y_i - y_{(i+1)}] > 0$
- *The sum I+V* This criterion results as a consequence of the shape of *I-V* characteristic
  The sum of the voltage value and the current value, for any of the observations, may not be less than the minimum between the value of the short-circuit current and the value of the open circuit voltage.

This results in the third criterion for refining the data::
Elimination of the data sets $\{x_i , y_i\}$ *for which* $(x_i+y_i)<min(I_{SC} , V_{OC})$

- *Averaging of the "end" values*

This criterion is specific to the experimental system presented in this work. The control mode and resolution of the ADC (12-bit) converters determine the collection of a large number of points near of the end of the interval ($V\approx max(V)$,$I\approx 0$) and ($V\approx 0$, $I\approx max(I)$).
This results in the fourth criterion for refining the data, which is at the same time the way of establishing the values for: I_{sc} and V_{oc}:
The value of the short-circuit current (Isc) and the value of the open circuit voltage (Voc) will be established by mediating the end values, based on a tolerance dependent on the characteristics of the experimental system (eg voltage divider ratio, converter resolution ADC, etc.).

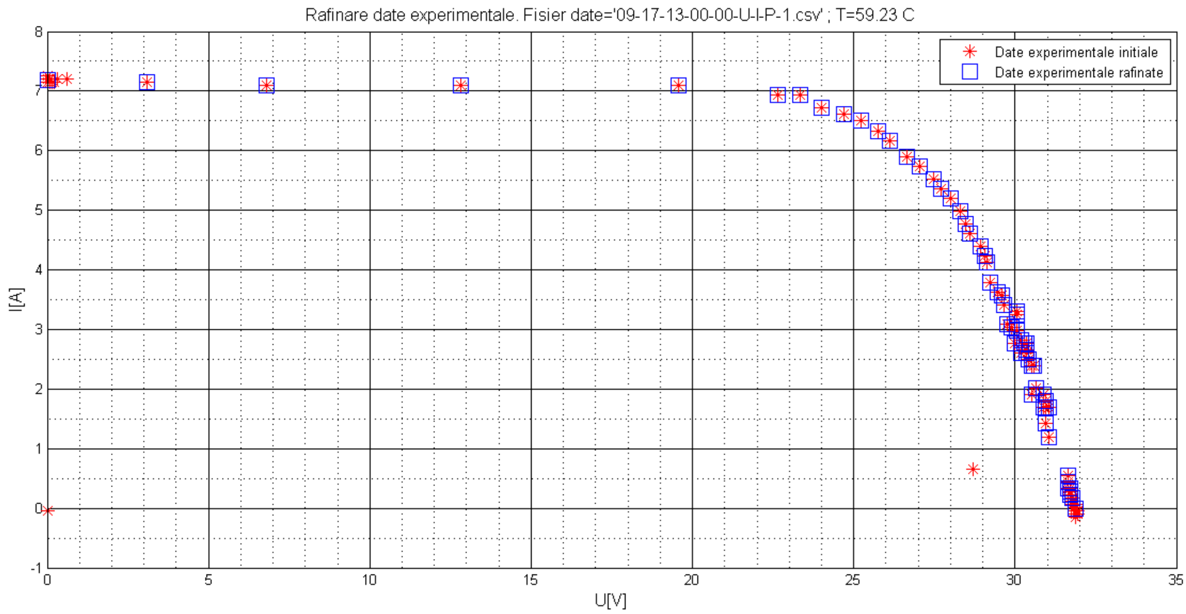


Fig. 9.2.1. *Experimental data after the refining process*
The values framed in squares are validated data
(Data refining tolerance: ± 2.5%)

Chapter 10
Validation of explicit equations based on experimental data

This chapter is intended to assess the approximation, based on experimental data, of the explicit equations proposed in this work.

The aim is to verify and confirm the usefulness of these equations in modeling of the experimental data obtained from the experimental system.

This chapter presents in detail the methodology for validating the explicit equation (3.2.17) and *Annex VI* presents data on the process for validating the equation (3.3.9).

For the validation of the explicit equation (3.2.17) data sets obtained with the experimental system presented in Chapter 7 were used.

The validation of equation (3.2.17) based on the experimental data was carried out in three stages:

- Stage 1. Determination of the parameters *m, Rs and Rp* based on experimental data using the explicit relation (3.2.17);
- Stage 2. Polynomial function identification for approximating of the experimental data;
- Stage 3. Comparison of the degree of approximation obtained based on the explicit relation with the degree of approximation obtained based on the polynomial function.

As example, in this chapter, the results of the performed simulations for a complete set of experimental data are presented. The values for the experimental data set used in this chapter are presented in *Annex V*.

This chapter highlights the usefulness of the data obtained, with the experimental system built, for the study of mathematical models approached in this field.

Chapter 11

Original contributions

The following are the contributions of the author, brought in the field of the thesis, in the order in which they appear in the content of this work.

Chapter 2

In this chapter is proposed and presented an original approach (new in the literature) on rewriting of the characteristic equations of photovoltaic cells in a normalized form. Normalized forms are proposed for all three models studied in this work.

The proposed normalized forms have advantages in terms of the simplicity of the relationships and in terms of the graphical representation of the characteristics.

Chapter 3

In this chapter is presented the mathematical demonstration after which the characteristic equations of the photovoltaic cell are converted from implicit-transcendent form into an explicit form. The demonstration is based on Lambert's W function and its properties in solving transcendental equations.

A number of simplifications of explicit equations are proposed. The purpose of these simplifications is to reduce the number of terms of the characteristic equations and to make their handling much easier.

The proposed simplified forms are analyzed in terms of accuracy by comparing the results of numerical simulations obtained on the basis of the initial form. These simulations showed that two of the proposed simplified shapes had a deviation of less than 15ppm. The third proposed form has a determination deviation of less than 0.2% which makes it sufficiently accurate for any type of evaluation of the characteristics of photovoltaic cells.

Chapter 4

In this chapter a very fast and precise iterative algorithm for calculating the values of Lambert's W function (positive branch) is proposed and presented. Determining the values of Lambert's W function is a necessary process when using the explicit forms of the equations of photovoltaic cell characteristics.

Based on this algorithm, a function (*Q_lambertw*) was implemented in MatLab, which ensures the extremely fast and precise processing of the values of Lambert's W function. The evaluation of the processing speed and the accuracy of the proposed functions is performed compared to the results obtained by using the “lambertw” function existing in the standard MatLab function package.

Based on the proposed function (*Q_lambertw*), a set of MatLab functions has been developed that ensures extremely fast processing of characteristics for photovoltaic cells. These functions ensure the evaluation of 1000 characteristics, each composed of 500 points, in less than 0.5msec. (MatLab R2016a installed on an Intel Core-i3-Gen4 platform).

<u>Chapter 5</u>

In this chapter, four sets of relations for calculation are proposed and presented

direct (non-iterative) of the coordinates of the maximum power point for the three-parameter model (3P- “ideal model”).

At least two of the relationships proposed in this work (5.1.34) (5.1.35) can be considered as the most accurate in the literature regarding the explicit-analytical determination of the coordinates of the maximum power point for the three-parameter model .

<u>Chapter 6</u>

In this chapter, three sets of relationships are proposed and presented for the direct (non-iterative) calculation of the coordinates of the maximum power point for the four-parameter model (4P).

Among the relations proposed in this chapter is the relation (6.1.46) which although has a relatively complex shape is characterized by a relative error in determining the current coordinate of the maximum power point (for the four-parameter model) less than 2 • 10 -6 (2ppm).

<u>Chapter 7</u>

In this chapter a new method is proposed, generically called the intersection method, to identify the position of the maximum power point on the voltage-current characteristic of photovoltaic cells.

The calculation ratios obtained by the method proposed in this chapter, ensure a relative error of less than 1% for estimating the maximum power for the intervals covering all real operating conditions for commercial photovoltaic cells (based on mono or poly-crystalline silicon) for a intensity of solar radiation greater than 200W / m2.

Chapter 8

This chapter makes an exhaustive description of an experimental system designed to "raise" the voltage-current characteristic for photovoltaic panels. The description shall include details of the principles underlying such a system, details of the design and calculation of an electronic load variation system and details of its execution.

The system simultaneously achieves the acquisition of voltage-current characteristic points, the operating temperature of photovoltaic cells and the acquisition of values of external environmental parameters (intensity of solar radiation, ambient temperature and wind speed).

The collected data were centralized in a database comprising a multitude of voltage-current characteristics for a wide range of real operating conditions of the measured panels.

Chapter 9

This chapter is devoted to the description of the process of refining the data obtained with the experimental system presented in this work.

A series of criteria for "critical analysis" and processing of the experimental data obtained are proposed, criteria that are established specifically for the experimental system presented.

Chapter 10

This chapter presents a methodology used to verify a mathematical model based on experimental.

This chapter presents a way to validate one of the explicit relationships presented in the work. For validation, a comparison is made between the results obtained by approximating the experimental data on a polynomial curve (degree 9) with the results obtained with the relationship proposed for validation.

This chapter highlights the usefulness of the data obtained, with the experimental system built, in the study of mathematical models approached in this field.

List of proposed relationships for calculating coordinates of the maximum power point of the photovoltaic cells

	I_{max}	V_{max}
3P Model	*(5.1.14)a*	*(5.1.14)b*
	(5.1.18)a	*(5.1.18)b*
	(5.1.34)a	*(5.1.34)b*
	(5.1.35)a	*(5.1.35)b*
4P Model	*(6.1.31)*	*(6.1.14)*
	(5.1.35)	*(6.1.14)*
	(5.1.44)	*(6.1.14)*
	(5.1.46)	*(6.1.14)*
5P Model	*(7.2.34.a)*	*(7.2.34.b)*

Articles published by the author in the doctoral thesis field

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